2017 Fall PHYS2310 電磁學（Electromagnetism) Midterm (double sides)

[Griffiths Chs. 1-3] 2017/11/14, 10:10am – 12:00am, 教師：張存續

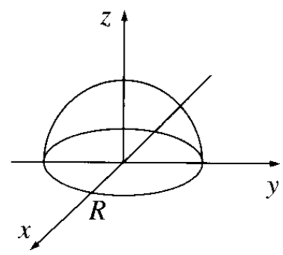
◇ Useful formulas $\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}v_\phi$$

1. (20%)

(a) Compute the divergence of the function $\mathbf{v} = r^2\cos\theta\hat{\mathbf{r}} + r^2\cos\phi\hat{\boldsymbol{\theta}} - r^2\cos\theta\sin\phi\hat{\boldsymbol{\phi}}$. (10%)

(b) Check the divergence theorem for this function, using as your volume the inverted hemispherical bowl of radius *R*, resting on the *xy* plane and centered at the origin (10%).



2. (20%)

(a) Prove that the normal component of **E** is discontinuous at any boundary, using Divergence theorem. (6%)

(b) Prove that the tangential component of **E** is always continuous, using Stokes' theorem. (6%)

(c) Write down the normal and tangential component of electric fields immediately outside a metal surface with the surface charge density σ. (8%)

3. (20%) The potential of some configuration is given by the expression $V(\mathbf{r}) = Ae^{-\lambda r}/r$, where *A* and λ are constants.

(a) Find the energy density. (6%)

(b) Find the charge density $\rho(\mathbf{r})$. (6%)

(c) Find the total charge *Q* (do it two different ways) and verify the divergence theorem. (8%)

4. (20%) Consider two concentric spherical shells, of radii a and b ($b>a$). The inner shell is connected to a potential $V(a,\theta)$ (to be given), while the outer shell is grounded $V(b,\theta)=0$.

(a) If $V(a,\theta)=V_0$ (constant), find the potential at $r<a$, $a<r<b$, and $r>b$. (10%)

(b) If $V(a,\theta)=V_0(1-\cos\theta-\cos^2\theta)$, find the potential everywhere between the shells ($a<r<b$). (10%) [Hint: use Legendre polynomials $P_0(x)=1$, $P_1(x)=x$, and $P_2(x)=(3x^2-1)/2$].

5. (20%) A sphere of radius R, centered at the origin, carries charge density

$$\rho(r,\theta)=k\frac{R}{r^2}(R-2r)\sin\theta,$$

where k is a constant, and r, θ are the usual spherical coordinates.

(a) Find the monopole, dipole, and quadrupole terms. (10%)

(b) Find the approximate potential for points on the z axis, far from the sphere. (10%)

[Hint: $V(\mathbf{r})=\frac{1}{4\pi\varepsilon_0}(\frac{1}{r}\int\rho(\mathbf{r}')d\tau'+\frac{1}{r^2}\int r'\cos\theta'\rho(\mathbf{r}')d\tau'+\frac{1}{r^3}\int(r')^2(\frac{3}{2}\cos^2\theta'-\frac{1}{2})\rho(\mathbf{r}')d\tau'+\ldots)$].